C. Hertling Mainz, 29.02.2016

Solution of <u>exercise 2</u>

(a) For $t \in \mathbb{C}$: $\Gamma(\mathbb{P}^1, \mathcal{H}(t))$ is generated by $v_1, \tau(v_1)$.

For $t \in \mathbb{P}^1 \setminus \{0\}$: $\Gamma(\mathbb{P}^1, \mathcal{H}(t))$ is generated by $t^{-1} v_1, \tau(t^{-1} v_1)$.

For $|t| = 1$: $\tau(v_1) = z \cdot \bar{t} \cdot v_1$, $\mathcal{H}(t) \cong \mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(1)$.

For $|t| \neq 1$: pure TERP structure, because $\begin{vmatrix} z^{-1} & t \\ \bar{t} & z \end{vmatrix} = 1 - |t|^2 \neq 0$

$$h(v_1, v_1) = P(v_1(z), \tau(v_1)(-z)) = P(z^{-1} A_1 + t A_2, (-z) A_2 + \bar{t} A_1) = -1 + t \cdot \bar{t}$$

$$h(v_1, \tau(v_1)) = P(v_1(z), v_1(-z)) = P(z^{-1} A_1 + t A_2, (-z)^{-1} A_1 + t A_2) = z^{-1} \cdot t \cdot 1 + t \cdot (-z) \cdot 1$$
$$= 0$$

$$h\left(\begin{pmatrix} v_1 \\ \tau(v_1) \end{pmatrix}, (v_1, \tau(v_1))\right) = (-1 + |t|^2) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

h positive definite outside S^1, h negative definite inside $S^1 \subset \mathbb{P}^1$.

$\mathcal{U}$ and $\mathcal{Q}$: $\nabla_{z\partial_z} (v_1, \tau(v_1)) = (-z^{-1} A_1, z A_2)$

$$= (v_1, \tau(v_1)) \cdot \begin{pmatrix} 1 & z \cdot \bar{t} \\ -z^{-1} \cdot t & -1 \end{pmatrix} \frac{1}{-1 + |t|^2}$$

$$= \left[\frac{1}{z} \mathcal{U} - \mathcal{Q} - z \cdot \tau \mathcal{U} \tau\right] (v_1, \tau(v_1))$$

$$= (v_1, \tau(v_1)) \cdot \left[\frac{1}{z} \mathcal{U}^{mat} - \mathcal{Q}^{mat} - z \cdot (\tau \mathcal{U} \tau)^{mat}\right]$$

$$= (v_1, \tau(v_1)) \cdot \left[\frac{1}{z} \begin{pmatrix} 0 & 0 \\ -t & 0 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - z \cdot \begin{pmatrix} 0 & \bar{t} \\ 0 & 0 \end{pmatrix}\right] \cdot \frac{1}{|t|^2 - 1}$$

The eigenvalues of $\mathcal{Q}^{mat}$ are $\frac{\pm 1}{|t|^2 - 1}$.

$\tau(v_1, \tau(v_1)) = (v_1, \tau(v_1)) \cdot \tau^{mat}$, $\tau^{mat} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

2)

(b) $$\tau\Big\downarrow\;\begin{array}{c|c|c|c|l} A_1 & A_2 & zA_2 & v_1 = z^{-1}A_1 + tA_2 & v_2 := t^{-1}z^{-1}A_1 + A_2 + \bar{t}^{-1} z A_1 \\ A_1 & A_2 & z^{-1}A_2 & \tau(v_1) = zA_1 + \bar{t}A_2 & \tau(v_2) = v_2 \end{array}$$

For $t \in \mathbb{P}^1 \setminus \{0\}$: the TERP structure is pure,
$\Gamma(\mathbb{P}^1, \mathcal{H}(t))$ is generated by A_1 and v_2.

For $t = 0$: the TERP structure is not pure,
$\Gamma(\mathbb{P}^1, \mathcal{H}(t))$ is generated by $z^{-1}A_1, A_1, zA_1$.
$\mathcal{H}(t) \cong \mathcal{O}_{\mathbb{P}^1}(-2) \oplus \mathcal{O}_{\mathbb{P}^1}(2)$.

For $t \in \mathbb{P}^1 \setminus \{0\}$:
$h\left(\begin{pmatrix} A_1 \\ v_2 \end{pmatrix}, (A_1, v_2)\right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, h has signature $(\overset{+}{1}, \overset{0}{0}, \overset{-}{1})$.

$\mathcal{U}$ and $\mathcal{Q}$:

$$\nabla_{z\partial_z}(A_1, v_2) = (0, -t^{-1} \cdot z^{-1} \cdot A_1 + \bar{t}^{-1} \cdot z \cdot A_1) = (A_1, v_2) \cdot \begin{pmatrix} 0 & -t^{-1}z^{-1} + \bar{t}^{-1}z \\ 0 & 0 \end{pmatrix}$$
$$= (A_1, v_2) \cdot \left[\frac{1}{z}\mathcal{U}^{mat} - \mathcal{Q}^{mat} - z(\tau\mathcal{U}\tau)^{mat}\right]$$
$$= (A_1, v_1) \cdot \left[\frac{1}{z}\begin{pmatrix} 0 & -t^{-1} \\ 0 & 0 \end{pmatrix} - 0 - z \cdot \begin{pmatrix} 0 & -\bar{t}^{-1} \\ 0 & 0 \end{pmatrix}\right]$$

$\tau^{mat} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.